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Reflection at non-free boundary of a micropolar piezoelectric half-space

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ABSTRACT

This paper presents a study on plane waves propagating in a micropolar piezoelectric material with transverse isotropy. The plane wave solutions of the governing field equations suggest that there exists three coupled plane waves propagating in a micropolar piezoelectric medium. Reflection of a plane wave incident at a non-free surface of a micropolar piezoelectric half-space is considered. The analytical expressions of the reflection coefficients and energy ratios are obtained. A quantitative example is set up to compute the speeds, reflection coefficients and energy ratios of reflected waves. The speeds, energy ratios and reflection coefficients are illustrated graphically against the propagation angle. The numerical results show that the boundary parameters connected to the mechanical nature of the non-free surface affect significantly the energy ratios and reflection coefficients.

1. Introduction

The elastic wave propagation in anisotropic media is basically distinguishable from their propagation in isotropic media. Due to the dependence of the elastic properties of anisotropic materials on the internal structure of the material, the wave characteristics in anisotropic media is of important concern in the fields like seismology, civil engineering and geophysics. In each direction of anisotropic elastic materials, there exist three body waves. These waves cannot in general be categorized into longitudinal and shear waves due to mutually perpendicular associated displacement vectors. A large number of analytical studies have elaborated the wave propagation in anisotropic media. Some prominent research papers on this phenomena are cited as Musgrave [1], Buchwald [2], Henneke [3], Keith and Crampin [4], Daley and Hron [5], Crampin [6,7], Levin [8], Rokhlin et al. [9], Carcione, et al. [10], Thomsen [11], Chadwick [12], Payton [13], Alshits et al. [14], Graebner [15], Chattopadhyay and Chaudhari [16], Carcione [17], Ruger [18], Singh and Khurana [19], Eskandari-Ghadi [20], Ting [21], Chattopadhyay [22], Chatterjee et al. [23] and Mondal et al. [24].

Wave propagation in piezoelectric materials has been used in creation and transmission of disturbances in transducers and resonators. The reflection and transmission of plane waves in piezoelectric materials have various applications in the field of transduction, signal processing and frequency control [25–29]. The plane waves in different piezoelectric materials have been explored by many researchers. Notable among them are Kyame [30], Pailloux [31], Hruska [32], Auld [33], Cheng and Sun [34], Alshits et al. [35], Every and Neiman [36], Alshits and Shuvalov [37] and various other researchers. During the past decade, the propagation of waves in piezoelectric materials has been the center of attraction. See for instances, Pang et al. [38] have computed the coefficients of reflection and refraction at piezoelectric/piezomagnetic interface. Darinskii et al. [39] have investigated the importance of electromagnetic waves during the reflection process in piezoelectric crystals. Burkov et al. [40] have investigated the uniaxial stress effect on reflected and refracted waves in piezoelectrics. Abd-alla and Al-sheikh [41] have studied the initial stress effect on the amplitude ratios of reflected and refracted waves at piezoelectric/peizoelectric interface. Vashishth and Gupta [42] have shown the piezoelectric effects on the plane waves in porous piezoelectric materials of transversely isotropic type. Singh [43] have investigated the pre-stress effects on reflection coefficients at an electrically shorted/charge-free and traction free boundary of a piezoelectric half-space. Kuang and Yuan [44] have examined some reflection and transmission problems in piezoelectric and pyroelectric medium. Yuan and Zhu [45] have obtained the amplitude ratios of reflected and refracted waves at a boundary separating two piezoelectric half-spaces. Abd-alla et al. [46] and Guo and Wei [47] have examined the influences of initial stresses on the amplitude ratios of reflected and refracted waves at an interface between two piezoelectric half-spaces.

In contrast to the classical elasticity, the micropolar theory of elasticity consider the local rotations with additional independent degrees of freedom. In micropolar theory, the displacement and microrotation vectors describe the particle motion. Eringen [48–50] introduced the linear micropolar elasticity which support the body couples in addition to body forces in micropolar materials. Various researchers have used the

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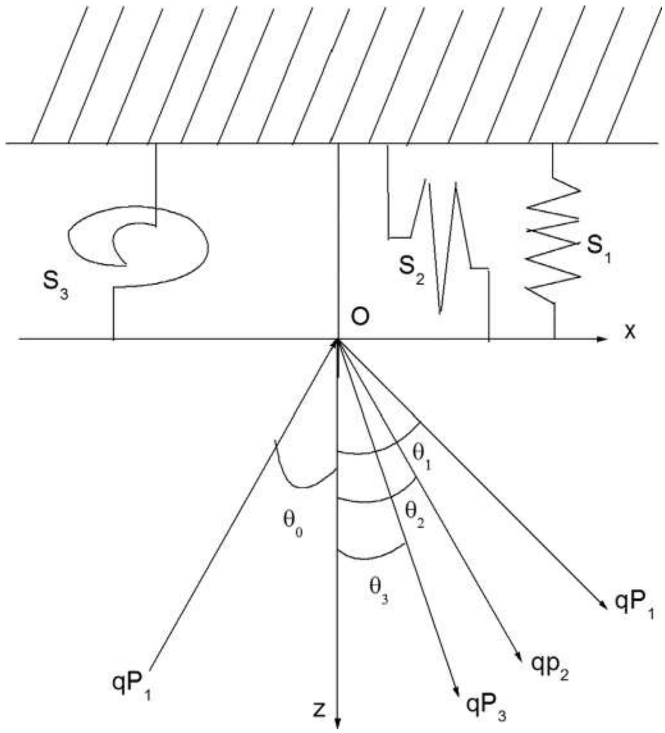

Fig. 1. Reflection from non-free surface of a micropolar piezoelectric solid half-space.

micropolar elasticity with the piezoelectricity. See for instances, Cracium [51] has considered the quasi-static electric fields to derive the basic equations of linear piezoelectric micropolar thermoelasticity. Vieru and Ciumasu [52] have studied the Love wave propagation along a layer of an isotropic dielectric micropolar material placed on a half-space of an isotropic micropolar material. Zhilin and Kolpakov [53] have proposed a micropolar theory of piezoelectric materials and have derived the field governing equations. Iesan [54] has proved a result for uniqueness and a theorem on reciprocity in context of linear theory of microstretch piezoelectricity. Aouadi [55] has investigated the dynamic micropolar piezoelectricity to establish a reciprocity relation. Gales [56] has formulated the initial boundary value problem and proved some uniqueness results in context of linear micromorphic piezoelectricity.

The coupling nature of the piezoelectric material makes it very important in fluid monitoring, sonar projecting, pulse generating and surface acoustic wave devices. The elastic wave propagation in a layered structure has many implementations in composite materials, geophysics, ocean acoustics, non-destructive evaluation and various other acoustic devices. Reflection coefficient is a useful tool for specifying the geoacoustic properties of the ocean floor and its sub-bottom structure. The reflection and refraction process in piezoelectric materials with effects of micropolarity has not been explored much till date. Some studies on wave propagation in piezoelectric materials with micropolarity have been cited herewith. Sangwan et al. [57] and Singh et al. [58] have studied the nature of the reflection and refraction coefficients at elastic/micropolar piezoelectric and micropolar piezoelectric/micropolar piezoelectric interfaces. Singh and Sindhu [59,60] have also derived the Rayleigh wave equations in a micropolar piezoelectric medium and have shown the effects of rotation and microrotation on wave speed of Rayleigh wave.

The boundary surface may be non-free with various constraints in actual engineering situations. Recently, Zhang et al. [61] has considered the reflection process of plane waves at a non-free surface of a micropolar elastic half-space. Motivated by the recent studies of Aouadi [55] and Zhang et al. [61], a problem on reflection of plane waves is considered at a non-free surface of a piezoelectric micropolar solid half-space. In Section 2, the governing equations for linear micropolar piezoelectric materials are specialized in x-z plane and are solved to explore the possible plane waves. In Section 3, the reflection phenomena for an incident plane is considered at non-free surface. The appropriate components of displacement, microrotation and electric potential for incident and reflected waves are formulated which satisfy the required surface conditions at the non-free surface. The Snell's law for the present half-space model and the relations in coefficients of reflection are obtained for an incident plane wave. In Section 4, the energy ratio expressions of reflected waves are also obtained. In Section 5, a quantitative example of the model is set up to graphically illustrate the effects of micropolarity and piezoelectricity on the ratios of amplitudes and energies of reflected waves. Finally, the theory and numerical analysis are concluded in last section.

2. Formulation of the problem and solution

We consider a homogeneous transversely isotropic micropolar piezoelectric half space. We take the origin of the coordinate system on the free surface and $z-$ axis is pointing normally into the half-space, which is thus represented by $z \geq 0$. We assume the components of the displacement and microrotation vectors of the form $\vec{u} = (u_1, 0, u_3)$ and $\vec{\phi} = (0, \phi_2, 0)$. Following Aouadi [55], the field equations governing the motions in $x - z$ plane are specialized as

$$A_{11}u_{1,11} + (A_{13} + A_{56})u_{3,13} + A_{55}u_{1,33} + K_1\phi_{2,3} - 2\lambda_{31}\psi_{,13} = \rho\ddot{u}_1, \tag{1}$$

$$A_{66}u_{3,11} + (A_{13} + A_{56})u_{1,13} + A_{33}u_{3,33} + K_2\phi_{2,1} - \lambda_{31}\psi_{,11} - \lambda_{33}\psi_{,33} = \rho\ddot{u}_3, \tag{2}$$

$$B_{77}\phi_{2,11} + B_{66}\phi_{2,33} - \chi\phi_2 - K_1u_{1,3} - K_2u_{3,1} = \rho\eta\ddot{\phi}_2, \tag{3}$$

$$\lambda_{31}u_{3,11} + \lambda_{33}u_{3,33} + 2\lambda_{31}u_{1,13} + \gamma_1\psi_{,11} + \gamma_3\psi_{,33} = 0, \tag{4}$$

where

$$K_1 = A_{56} - A_{55}, \quad K_2 = A_{66} - A_{56}, \quad \chi = K_2 - K_1.$$

Now, we seek the two dimensional solutions of Eqs. (1)–(4) in following form

$$\{u_1, u_3, \phi_2, \psi\} = \{\bar{u}_1, \bar{u}_3, \bar{\phi}_2, \bar{\psi}\}\exp\{\iota k(x\sin\theta + z\cos\theta - vt)\} \tag{5}$$

where k is wave number, v is phase velocity and $\bar{u}_1, \bar{u}_3, \bar{\phi}_2, \bar{\psi}$ are arbitrary constants. With the help of Eq. (5), the Eqs. (1)–(4) admit the non-trivial solutions under the following condition

$$\begin{vmatrix} D_1-\zeta & L_1 & K_1\cos\theta & L_2 \\ L_1 & D_2-\zeta & K_2\sin\theta & D_3 \\ K_1^*\cos\theta & K_2^*\sin\theta & D_4-\zeta & 0 \\ L_2 & D_3 & 0 & -D_5 \end{vmatrix} = 0 \tag{6}$$

where

$$D_1 = A_{11}\sin^2\theta + A_{55}\cos^2\theta, \quad D_2 = A_{66}\sin^2\theta + A_{33}\cos^2\theta,$$
$$D_3 = \lambda_{31}\sin^2\theta + \lambda_{33}\cos^2\theta,$$

$$D_4 = \frac{B_{77}\sin^2\theta + B_{66}\cos^2\theta}{j} + \frac{\chi}{jk^2},$$
$$D_5 = \gamma_{11}\sin^2\theta + \gamma_{33}\cos^2\theta, \quad \zeta = \rho v^2,$$

$$L_1 = (A_{13} + A_{56})\sin\theta\cos\theta, \quad L_2 = 2\lambda_{31}\sin\theta\cos\theta,$$
$$K_1^* = \frac{K_1}{\eta k^2}, \quad K_2^* = \frac{K_2}{\eta k^2}.$$

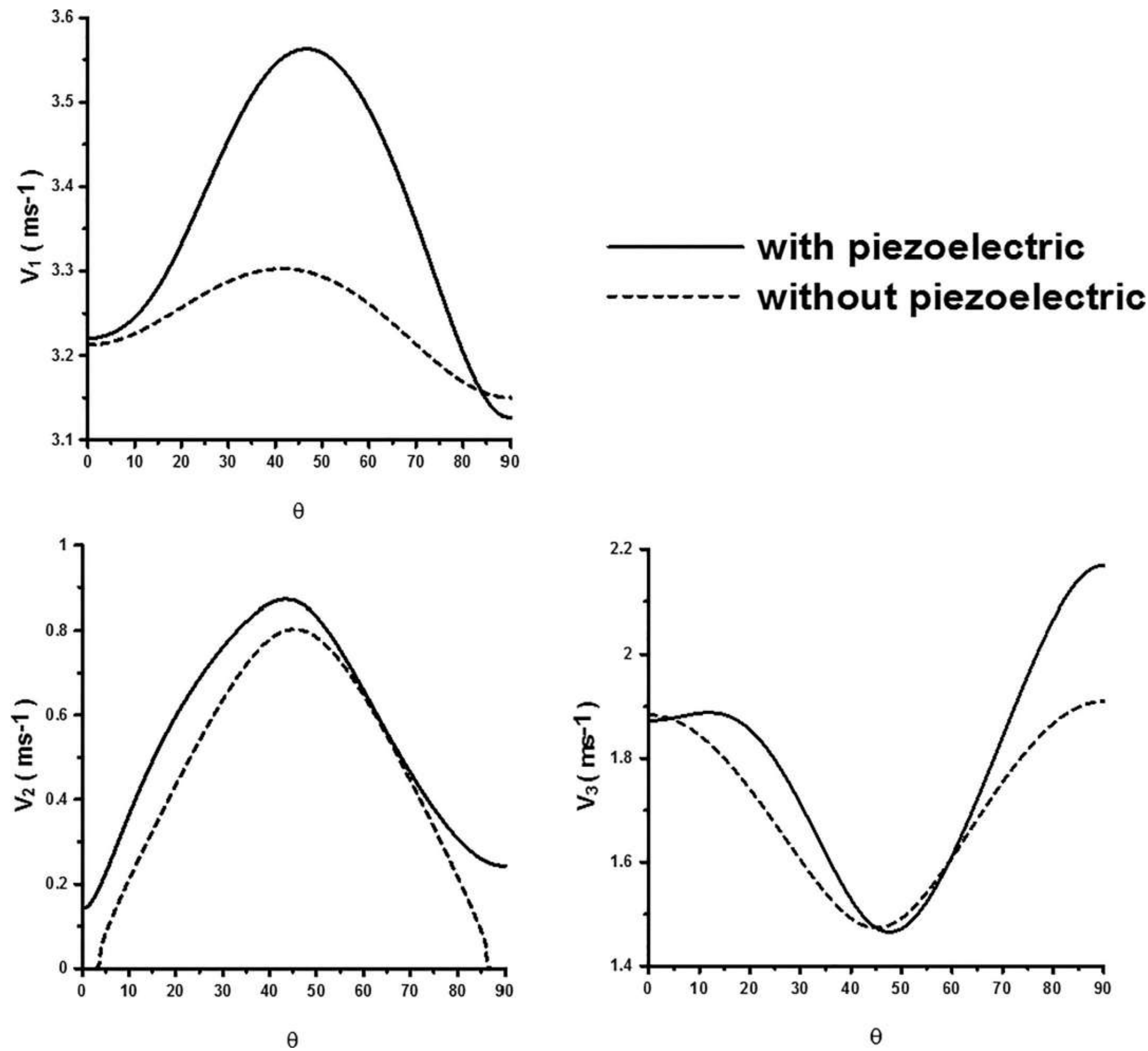

Fig. 2. Variations of the phase speeds V_1, V_2 and V_3 of plane waves qP_1, qP_2 and qP_3 against the propagation angle θ.

Eq. (6) is a cubic equation in ζ, which has been solved numerically by using Cardano method. The three roots $\zeta_1 = \rho\, \upsilon_1^2, \zeta_2 = \rho\, \upsilon_2^2$, and $\zeta_3 = \rho\, \upsilon_3^2$, correspond to three quasi plane waves, namely, qP_1, qP_2 and qP_3 waves. The square of complex phase velocities $\upsilon_j{}^2, (j = 1, 2, 3)$ of quasi plane waves will change with the phase propagation direction. Then, the complex phase velocities of the quasi-waves ($\upsilon_j = r_j + i s_j$) defines phase propagation velocities $V_j = (r_j^2 + s_j^2)/r_j$ and attenuation quality factors $q_j = -2s_j/r_j$ for each j. Therefore, three propagating waves in micropolar piezoelectric medium are attenuating waves. For real slowness vector, the direction of attenuation is along the direction of propagation. Hence, these waves are called as homogeneous waves.

3. Reflection from a non-free surface

In this section, the reflection of qP_1 wave from a non-free surface $z = 0$ is considered. For an incident qP_1 wave propagating through half-space, the qP_1, qP_2 and qP_3 waves get reflected back into the half-space. The complete geometry depicting the directions of the incident and reflected waves is given in Fig. 1. Following Zhang et al. [61], the required surface conditions at a non-free surface at $z = 0$ are

$$\sigma_{33} = -\iota\, S_1 u_3, \quad \sigma_{31} = -\iota\, S_2 u_1, \quad m_{32} = -\iota\, S_3 \phi_2. \tag{7}$$

where the proportional coefficients S_1, S_2 and S_3 are the stiffness of the normal, tangent and rotational elastic support, and

$$\sigma_{31} = A_{55} u_{1,3} + A_{56} u_{3,1} + K_1 \phi_2 - \lambda_{31} \psi_{,1}, \tag{8}$$

$$\sigma_{33} = A_{13} u_{1,1} + A_{33} u_{3,3} - \lambda_{33} \psi_{,3}, \tag{9}$$

$$m_{32} = B_{66} \phi_{2,3}. \tag{10}$$

The appropriate displacement components, microrotation component and electric potential for incident and reflected waves are

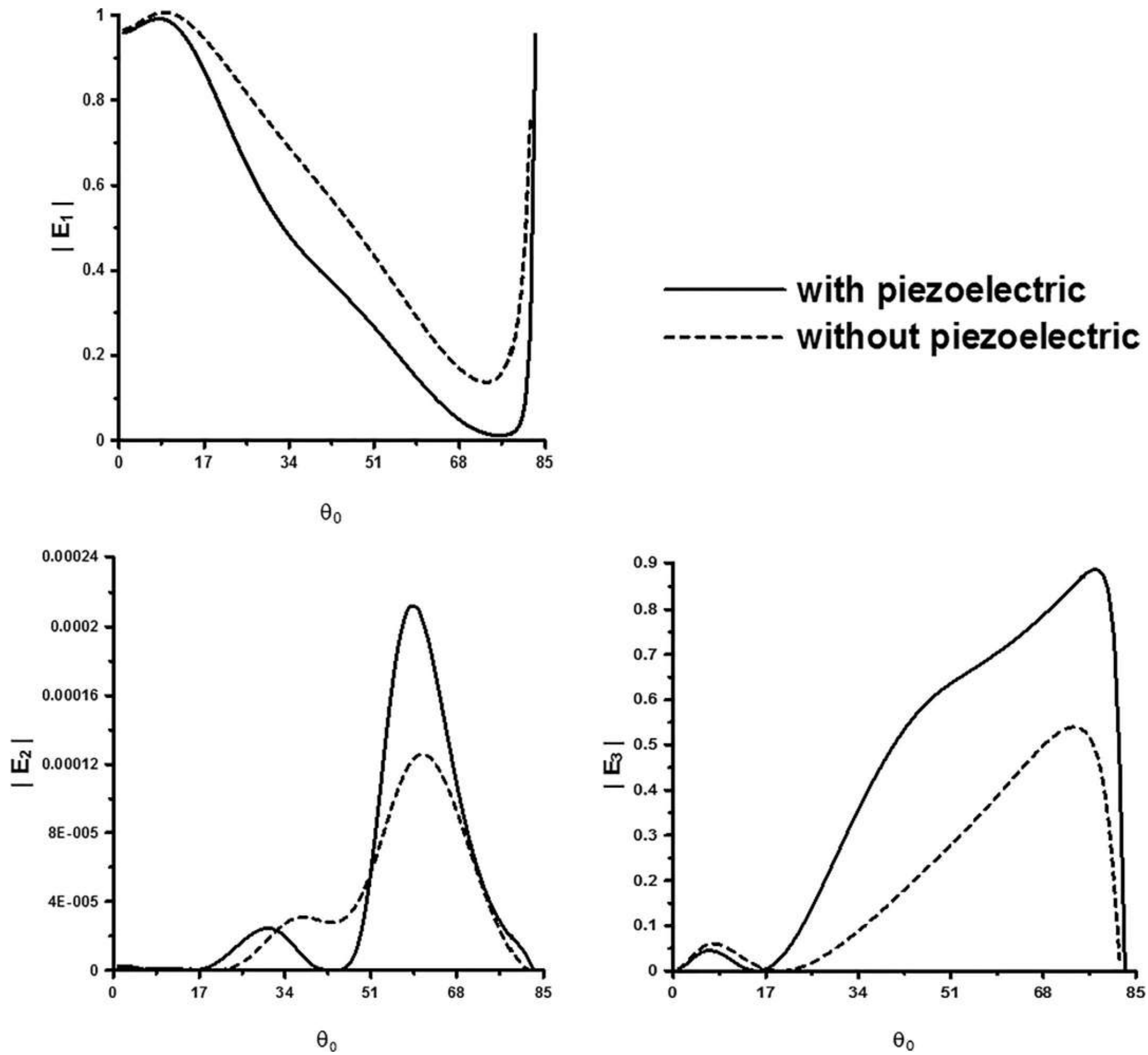

Fig. 3. Piezoelectric effects on the energy ratios $|E_1|, |E_2|$ and $|E_3|$ of reflected qP_1, qP_2 and qP_3 waves from a free boundary for an incident qP_1 wave.

$$(u_1, u_3, \phi_2, \psi) = (1, \xi_0, \eta_0, \nu_0)A_0 \exp\{\iota\, k_1(x \sin\theta_0 + z \cos\theta_0 - V_1 t)\}$$

$$+ \sum_{j=1}^{3} [(1, \xi_j, \eta_j, \nu_j) A_j \exp\{\iota\, k_j (x \sin\theta_j + z \cos\theta_j - V_j t)\}] \tag{11}$$

where the expressions for coupling coefficients of ξ_j, η_j, ν_j (j = 0, 1, 2, 3) are given in Appendix. The solutions given by (11) satisfy the boundary conditions (7) under the following Snell's law

$$k_1 V_1 = k_2 V_2 = k_3 V_3, \quad \frac{\sin\theta_0}{V_1} = \frac{\sin\theta_j}{V_j}, \; (j = 1, 2, 3) \tag{12}$$

and we obtain the following non-homogeneous system of three equations in reflection coefficients

$$\sum_{j=1}^{3} a_{ij} Z_j = b_i, \quad (i = 1, 2, 3) \tag{13}$$

and

$$Z_1 = \frac{A_1}{A_0}, \quad Z_2 = \frac{A_2}{A_0}, \quad Z_3 = \frac{A_3}{A_0}$$

are the reflection coefficients of the reflected qP_1, qP_2 and qP_3 waves, respectively, and

$$a_{1j} = \left\{ A_{13} \sin\theta_j + \xi_j \left(\frac{S_1}{k_j} - A_{33} \cos\theta_j \right) + \nu_j \lambda_{33} \cos\theta_j \right\} \left(\frac{k_j}{k_1} \right), \quad b_1 = -a_{11},$$

$$a_{2j} = \left\{ \xi_j A_{56} \sin\theta_j - A_{55} \cos\theta_j + \frac{S_2}{k_j} - \iota\, K_1 \frac{\eta_j}{k_j} - \nu_j \lambda_{31} \sin\theta_j \right\} \left(\frac{k_j}{k_1} \right),$$

$$b_2 = -\left\{ A_{55} \cos\theta_0 + \xi_0 A_{56} \sin\theta_0 + \frac{S_2}{k_1} - \iota\, K_1 \frac{\eta_0}{k_1} - \nu_0 \lambda_{31} \sin\theta_0 \right\},$$

$$a_{3j} = \frac{\eta_j}{k_j} \left(\frac{S_3}{k_j} - B_{66} \cos\theta_j \right) \left(\frac{k_j}{k_1} \right)^2, \quad b_3 = -\frac{\eta_0}{k_1} \left(\frac{S_3}{k_1} + B_{66} \cos\theta_0 \right).$$

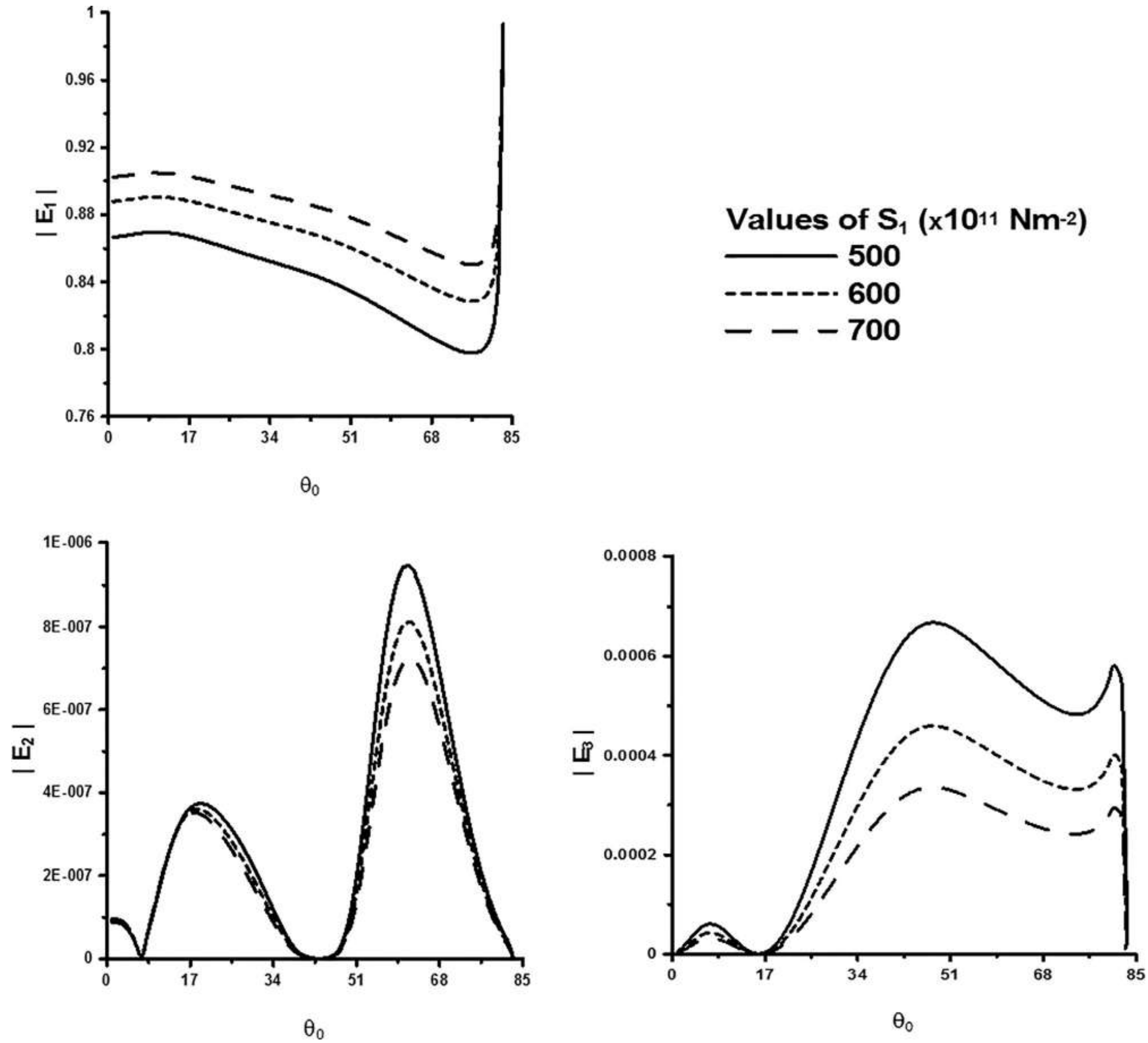

Fig. 4. Effects of normal stiffness coefficient on the energy ratios $|E_1|, |E_2|$ and $|E_3|$ of reflected qP_1, qP_2 and qP_3 waves from a non-free boundary for an incident qP_1 wave.

4. Energy ratios

Following Achenbach [62], the time average rate of energy transmission per unit surface area is given by

$$< P^* >= \sigma_{33}\dot{u}_3 + \sigma_{31}\dot{u}_1 + m_{32}\dot{\phi}_2, \tag{14}$$

Using (11) into (14), we obtain the following expressions for energy ratios of reflected qP_1, qP_2 and qP_3 waves

$$|E_j| = \left(\frac{k_j}{k_1}\right)^2\left(\frac{V_j}{V_1}\right)\left\{\frac{X_j \sin\theta_j - Y_j \cos\theta_j - \iota K_1(\eta_j/k_j)}{X_0 \sin\theta_1 + Y_0 \cos\theta_1 - \iota K_1(\eta_0/k_1)}\right\}Z_j^2, \quad (j = 1, 2, 3). \tag{15}$$

where

$$X_0 = \xi_0(A_{13} + A_{56}) - \nu_0\lambda_{31}, \quad Y_0 = \xi_0^2 A_{33} + A_{55} - \xi_0\nu_0\lambda_{33} + B_{66}\eta_0^2.$$

$$X_j = \xi_j(A_{13} + A_{56}) - \nu_j\lambda_{31}, \quad Y_j = \xi_j^2 A_{33} + A_{55} - \xi_j\nu_j\lambda_{33} + B_{66}\eta_j^2.$$

In the absence of piezoelectric parameters, i.e. if we take $\lambda_{31} = \lambda_{33} = \gamma_1 = \gamma_3$, the above analysis will reduce for the case of transversely isotropic micropolar elastic material. Further, in absence of transverse isotropy, these results agree with Zhang et al. [61]. Also, in the absence of micropolar parameters i.e. if we take $K_1 = K_2 = \chi = B_{66} = B_{77} = 0$, the above analysis will reduce for the case of transversely isotropic piezoelectric material.

5. Numerical results and discussion

Due to the non-availability of the experimental data of a transversely isotropic micropolar piezoelectric material, the numerical simulations/solutions of the present problem are restricted for an arbitrary quantitative example. For the purpose of numerical computations of the speeds and energy ratios of reflected waves, the following relevant physical constants are taken [57–60]:

$A_{11} = 17.8 \times 10^{11}$ Nm^{-2}, $A_{33} = 18.43 \times 10^{11}$ Nm^{-2}, $A_{13} = 7.59 \times 10^{11}$ Nm^{-2}, $A_{56} = 1.89 \times 10^{11}$ Nm^{-2}, $A_{55} = 4.357 \times 10^{11}$ Nm^{-2}, $A_{66} = 4.42 \times 10^{11}$ Nm^{-2}, $B_{77} = 0.278 \times 10^{10}$ N, $B_{66} = 0.268 \times 10^{10}$ N, $\lambda_{31} = 12$Cm^{-2}, $\lambda_{33} = 1.33$Cm^{-2}, $\gamma_1 = 85.2$C^2 N^{-1} m^{-2}, $\gamma_3 = 28.7$C^2 N^{-1} m^{-2}, $\rho = 1.74 \times 10^3$ Kg m^{-3}, j= 0.196 m^2.

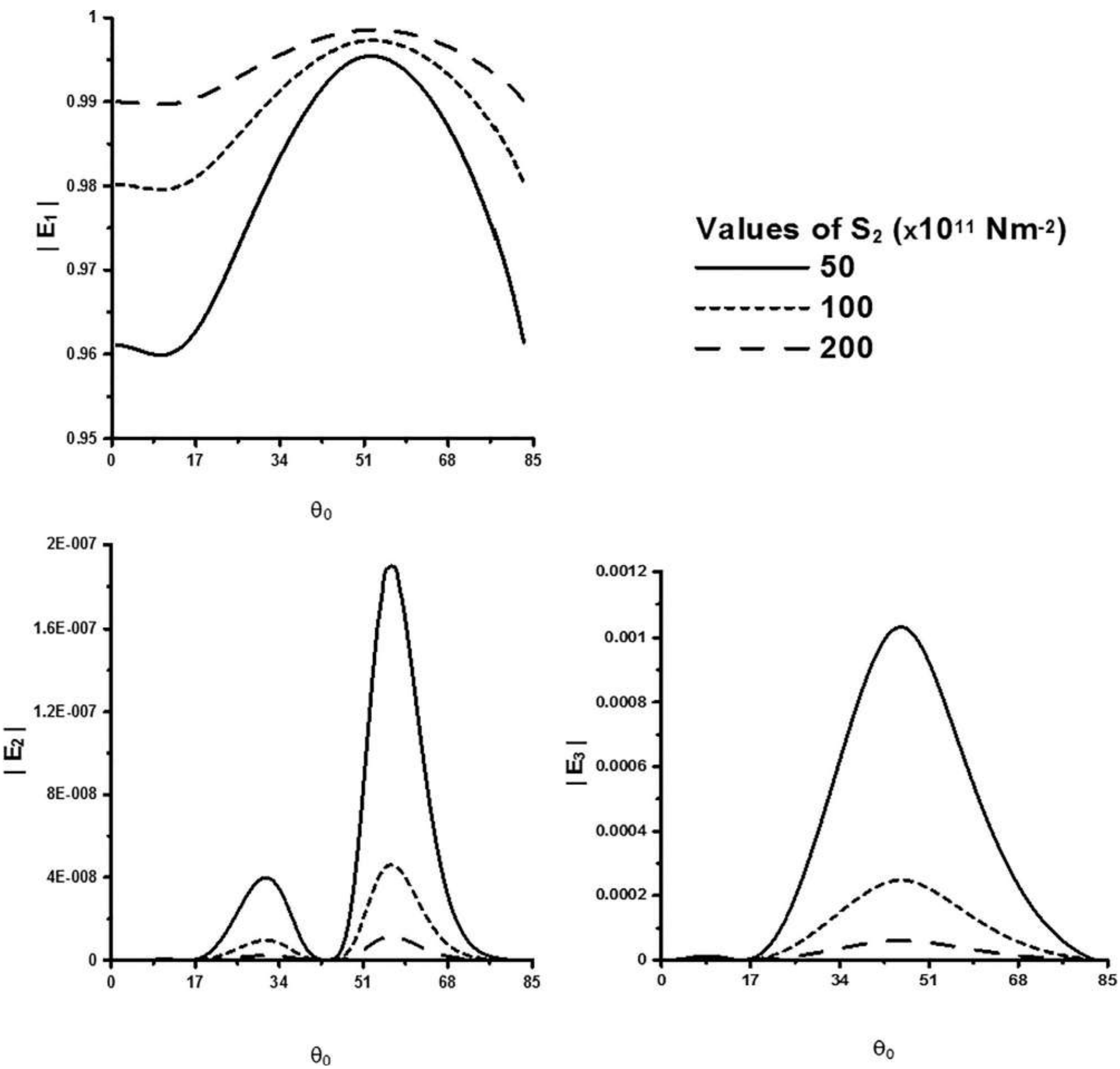

Fig. 5. Effects of tangential stiffness coefficient on the energy ratios $|E_1|, |E_2|$ and $|E_3|$ of reflected qP_1, qP_2 and qP_3 waves from a non-free boundary for an incident qP_1 wave.

For above physical parameters, the plane wave speeds and the energy ratios of reflected waves are plotted in Figs. 2–6 against the angle of propagation/incidence.

Fig. 2 illustrates the speeds V_1, V_2 and V_3 of three plane waves against the angle of propagation both in presence and absence of piezoelectric parameters. In presence of piezoelectric parameters, the value of speed V_1 of qP_1 wave is $3.22026 \times 10^4 \mathrm{m\,s}ww^{-1}$ at $\theta = 0^o$ and it increases monotonically to its maximum value $3.56299 \times 10^4 \mathrm{m\,s}^{-1}$ at $\theta = 47^o$ and then decreases to a value $3.12626 \times 10^4 \mathrm{m\,s}^{-1}$ at $\theta = 90^o$. A similar variation of the speed V_2 against θ has been observed for qP_2 wave. It first increases to its maximum value at $\theta = 43^o$ and then decreases till $\theta = 90^o$. However, the variation of the speed V_3 of qP_3 wave against θ has been observed converse to those noted for qP_1 and qP_2 waves. In absence of piezoelectric parameters, the solid speed variations in Fig. 2 reduce to the dotted variations. The piezoelectric parameters affect the speeds in a different manner at different propagation angle.

Fig. 3 demonstrates the energy ratios $|E_1|, |E_2|$ and $|E_3|$ of reflected qP_1, qP_2 and qP_3 waves from free boundary against the angle of incidence $\theta_0(1^o - 83^o)$ for both in presence and absence of piezoelectric parameters. For angles of incidence greater than 83^o, the reflected waves do not exist for present quantitative example of material. In presence of piezoelectric parameters, the energy ratio $|E_1|$ of qP_1 wave is found to be 0.9587 at $\theta_0 = 1^o$ and it decreases sharply and monotonically to its minimum value 0.0111 at at $\theta_0 = 76^o$ and then increases sharply to a value 0.9551 at $\theta_0 = 83^o$. However, the variations of energy ratios $|E_2|$ and $|E_3|$ are observed opposite to those for $|E_1|$ and attain their respective maximum values at $\theta_0 = 59^o$ and $\theta_0 = 78^o$. The energy ratio sum of all three reflected waves at each incident angle remains less than unity. In absence of piezoelectric parameters, the solid energy ratio variations in Fig. 3 reduce to dotted variations. The effect of piezoelectric parameters on the energy ratios has been noticed at each angle of incidence in the given range.

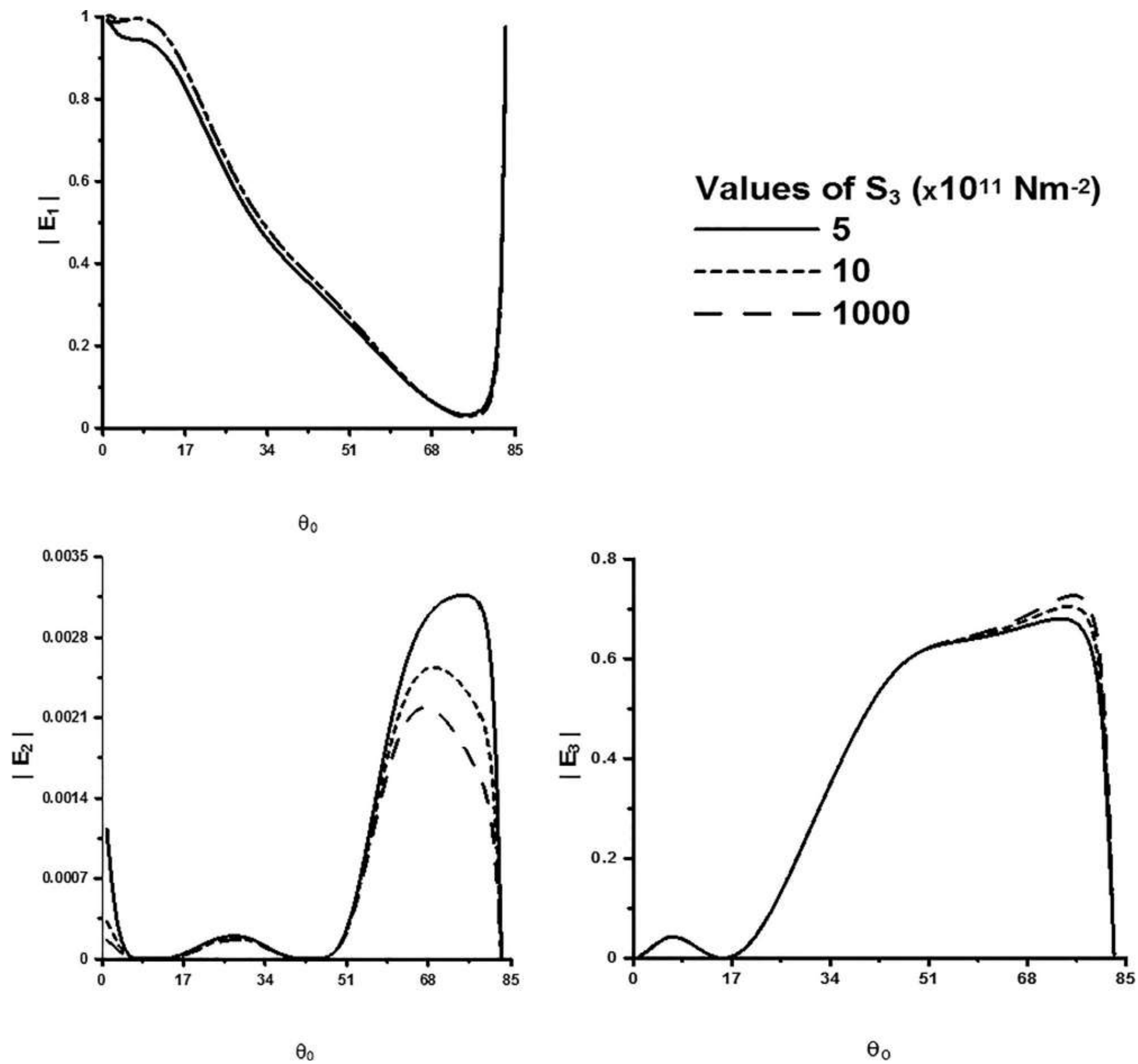

Fig. 6. Effects of rotational stiffness coefficient on the energy ratios $|E_1|$, $|E_2|$ and $|E_3|$ of reflected qP_1, qP_2 and qP_3 waves from a non-free boundary for an incident qP_1 wave.

Figs. 4 –6 have been plotted to show the effect of non-free boundary on the energy ratios of reflected waves. The Fig. 4 illustrates the effects of normal stiffness coefficient S_1 on the energy ratios of reflected waves at each angle of incidence. It has been noticed that the increase in value of S_1 enhances the energy ratio $|E_1|$ and drops the energy ratios $|E_2|$ and $|E_3|$ at a particular incident angle. The tangential and rotational stiffness coefficients (S_2 and S_3) also affect considerably the energy ratios of reflected waves and are shown in Figs. 5 and 6, respectively. If we compare the curves in Figs. 4–6 with solid curves in Fig. 3, the difference in free and non-free case is clearly visible. Therefore, by considering boundary as non-free, we have obtained different reflection characteristics in a micropolar piezoelectric medium.

6. Conclusion

A problem on plane waves in a transversely isotropic piezoelectric micropolar medium has been studied in this paper. The solutions of governing equation indicates the existence of three quasi plane waves. Reflection phenomena of plane waves from a non-free boundary has been considered. The relations and expressions for reflection coefficients and energy ratios have been derived. The experimental values of physical constants of a transversely isotropic micropolar piezoelectric material are not available in literature yet. Therefore, the numerical simulations of the wave characteristics are restricted to a quantitative example with arbitrary physical constants. The piezoelectric effects has been observed on the speeds of plane waves and energy ratio of reflected waves. The energy ratios have also been influenced by the normal, tangential and rotational stiffness coefficients. Numerical simulations/solutions of wave characteristics will definitely vary by taking other arbitrary values of the physical constants. However, it will be more interesting to discuss/solve some more numerical examples with the availability of the experimental physical constants of transversely isotropic micropolar piezoelectric materials.

Declaration of Competing Interest

This piece of the submission is being sent via mail.

Appendix

Making use of Eq. (11) in Eqs. (1)–(3), the expressions for coupling coefficients ξ_j, η_j, ν_j (j = 0, 1, 2, 3) are obtained as

$$\xi_j = \frac{M^{(j)}(\rho v_j^2 - D_4^{(j)}) + L^{(j)}K_1^* \cos\theta_j}{N^{(j)}(\rho v_j^2 - D_4^{(j)}) + L^{(j)}K_2^* \sin\theta_j}, \tag{16}$$

$$\imath\eta_j/k_j = \frac{-K_1^* \cos\theta_j + \xi_j K_2^* \sin\theta_j}{D_4^{(j)} - \rho v_j^2}, \tag{17}$$

$$\nu_j = \frac{L^{(j)}\xi_j + (\imath\eta_j/k_j)K_1 \cos\theta_j + D_1^{(j)} - \rho v_j^2}{L_2^{(j)}}, \tag{18}$$

$$\xi_0 = -\xi_1, \quad \eta_0 = -\eta_1, \quad \nu_0 = -\nu_1, \tag{19}$$

where
$L^{(j)} = -D_3^{(j)}K_1\cos\theta_j - L_2^{(j)}K_2\sin\theta_j$, $M^{(j)} = D_3^{(j)}(\rho v_j^2 - D_1^{(j)}) + L_1^{(j)}L_2^{(j)}$,
$N^{(j)} = L_2^{(j)}(\rho v_j^2 - D_2^{(j)}) - L_1^{(j)}D_3^{(j)}$.

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